What You’ll Learn

► You will discover that many reactions and processes reach a state of equilibrium.

► You will use Le Châtelier’s principle to explain how various factors affect chemical equilibria.

► You will calculate equilibrium concentrations of reactants and products using the equilibrium constant expression.

► You will determine the solubilities of sparingly soluble ionic compounds.

Why It’s Important

The concentrations of substances called acids and bases in your blood are crucial to your health. These substances continuously enter and leave your bloodstream, but the chemical equilibria among them maintain the balance needed for good health.

Visit the Chemistry Web site at chemistrymc.com to find links about chemical equilibrium.
DISCOVERY LAB

What's equal about equilibrium?

Does equilibrium mean that the amounts of reactants and products are equal?

Safety Precautions

Always wear safety goggles and a lab apron.

Procedure

1. Measure 20 mL of water in a graduated cylinder and pour it into a 100-mL beaker. Fill the graduated cylinder to the 20-mL mark. Place a glass tube in the graduated cylinder and another glass tube in the beaker. The tubes should reach the bottoms of the containers.

2. Cover the open ends of both glass tubes with your index fingers. Simultaneously, transfer the water from the cylinder to the beaker, and from the beaker to the cylinder.

3. Repeat the transfer process about 25 times. Record your observations.

Analysis

How can you explain your observations during the transfer process? What does this tell you about the concept of equilibrium?

Objectives

• Recognize the characteristics of chemical equilibrium.

• Write equilibrium expressions for systems that are at equilibrium.

• Calculate equilibrium constants from concentration data.

Vocabulary

reversible reaction
chemical equilibrium
law of chemical equilibrium
equilibrium constant
homogeneous equilibrium
heterogeneous equilibrium

When you get off a whirling amusement park ride, you probably pause a minute to “get your equilibrium.” If so, you are talking about getting your balance back after the ride exerted rapidly changing forces on you. But soon you are balanced steadily on your feet once more. Often, chemical reactions also reach a point of balance or equilibrium. The DISCOVERY LAB is an analogy for chemical equilibrium. You found that a point of balance was reached in the transfer of water from the beaker to the graduated cylinder and from the graduated cylinder to the beaker.

What is equilibrium?

Consider the reaction for the formation of ammonia from nitrogen and hydrogen that you learned about in Chapter 16.

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightarrow 2\text{NH}_3(g) \quad \Delta G^\circ = -33.1 \text{ kJ} \]

This reaction is important to agriculture because ammonia is used widely as a source of nitrogen for fertilizing corn and other farm crops. The photo on the opposite page shows ammonia being “knifed” into the soil.

Note that the equation for the production of ammonia has a negative standard free energy, \( \Delta G^\circ \). Recall that a negative sign for \( \Delta G^\circ \) indicates that the
reaction is spontaneous under standard conditions. Standard conditions are defined as 298 K and one atmosphere pressure. But spontaneous reactions are not always fast. When carried out under standard conditions, this ammonia-forming reaction is much too slow. To produce ammonia at a rate that is practical, the reaction must be carried out at a much higher temperature than 298 K and a higher pressure than one atmosphere.

What happens when one mole of nitrogen and three moles of hydrogen, the amounts shown in the equation, are placed in a closed reaction vessel at 723 K? Because the reaction is spontaneous, nitrogen and hydrogen begin to react. Figure 18-1 illustrates the progress of the reaction. Note that the concentration of the product, NH₃, is zero at the start and gradually increases with time. The reactants, H₂ and N₂, are consumed in the reaction, so their concentrations gradually decrease. After a period of time, however, the concentrations of H₂, N₂, and NH₃ no longer change. All concentrations become constant, as shown by the horizontal lines on the right side of the diagram. The concentrations of H₂ and N₂ are not zero, so not all of the reactants were converted to product even though \( \Delta G^\circ \) for this reaction is negative.

**Reversible reactions** When a reaction results in almost complete conversion of reactants to products, chemists say that the reaction goes to completion. But most reactions, including the ammonia-forming reaction, do not go to completion. They appear to stop. The reason is that these reactions are reversible. A **reversible reaction** is one that can occur in both the forward and the reverse directions.

\[
\text{Forward: } \text{N}_2(g) + 3\text{H}_2(g) \rightarrow 2\text{NH}_3(g) \\
\text{Reverse: } \text{N}_2(g) + 3\text{H}_2(g) \leftarrow 2\text{NH}_3(g)
\]

Chemists combine these two equations into a single equation that uses a double arrow to show that both reactions occur.

\[
\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)
\]

When you read the equation, the reactants in the forward reaction are on the left. In the reverse reaction, the reactants are on the right. In the forward reaction, hydrogen and nitrogen combine to form the product ammonia. In the reverse
reaction, ammonia decomposes into the products hydrogen and nitrogen. How does the reversibility of this reaction affect the production of ammonia?

**Figure 18-2a** shows a mixture of nitrogen and hydrogen just as the reaction begins at a definite, initial rate. No ammonia is present so only the forward reaction can occur.

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightarrow 2\text{NH}_3(g) \]

As hydrogen and nitrogen combine to form ammonia, their concentrations decrease, as shown in **Figure 18-2b**. Recall from Chapter 17 that the rate of a reaction depends upon the concentration of the reactants. The decrease in the concentration of the reactants causes the rate of the forward reaction to decrease. As soon as ammonia is present, the reverse reaction can occur, slowly at first, but at an increasing rate as the concentration of ammonia increases.

\[ \text{N}_2(g) + 3\text{H}_2(g) \leftarrow 2\text{NH}_3(g) \]

As the reaction proceeds, the rate of the forward reaction continues to decrease and the rate of the reverse reaction continues to increase until the two rates are equal. At that point, ammonia is being produced as fast as it is being decomposed, so the concentrations of nitrogen, hydrogen, and ammonia remain constant, as shown in **Figures 18-2c** and 18-2d. The system has reached a state of balance or equilibrium. The word *equilibrium* means that opposing processes are in balance. **Chemical equilibrium** is a state in which the forward and reverse reactions balance each other because they take place at equal rates.

\[ \text{Rate}_{\text{forward reaction}} = \text{Rate}_{\text{reverse reaction}} \]
You can recognize that the ammonia-forming reaction reaches a state of chemical equilibrium because its chemical equation is written with a double arrow like this.

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g) \]

At equilibrium, the concentrations of reactants and products are constant, as you saw in Figures 18-2c and 18-2d. However, that doesn’t mean that the amounts or concentrations of reactants and products are equal. That is seldom the case. In fact, it’s not unusual for the equilibrium concentrations of a reactant and product to differ by a factor of one million or more.

**The dynamic nature of equilibrium** Equilibrium is a state of action, not inaction. For example, consider this analogy. The Golden Gate Bridge, shown in Figure 18-3, connects two California cities, San Francisco and Sausalito. Suppose that all roads leading into and out of the two cities are closed for a day—except the Golden Gate Bridge. In addition, suppose that the number of vehicles per hour crossing the bridge in one direction equals the number of vehicles per hour traveling in the opposite direction. Given these circumstances, the number of vehicles in each of the two cities remains constant even though vehicles continue to cross the bridge. In this analogy, note that the total numbers of vehicles in the two cities do not have to be equal. Equilibrium requires only that the number of vehicles crossing the bridge in one direction is equal to the number crossing in the opposite direction.

The dynamic nature of chemical equilibrium can be illustrated by placing equal masses of iodine crystals in two interconnected flasks, as shown in Figure 18-4a. The crystals in the flask on the left contain iodine molecules made up entirely of the nonradioactive isotope I-127. The crystals in the flask on the right contain iodine molecules made up of the radioactive isotope I-131. The Geiger counters indicate the radioactivity within each flask.
Each flask is a closed system. No reactant or product can enter or leave. At 298 K and one atmosphere pressure, this equilibrium is established in both flasks.

\[ \text{I}_2(\text{s}) \rightleftharpoons \text{I}_2(\text{g}) \]

In the forward process, called sublimation, iodine molecules change directly from the solid phase to the gas phase. In the reverse process, gaseous iodine molecules return to the solid phase. A solid-vapor equilibrium is established in each flask.

When the stopcock in the tube connecting the two flasks is opened, iodine vapor can travel back and forth between the two flasks. After a period of time, the readings on the Geiger counters indicate that the flask on the left contains as many radioactive I-131 molecules as the flask on the right in both the vapor and the solid phases. See Figure 18-4b. How could radioactive I-131 molecules that were originally in the crystals in the flask on the right become part of the crystals in the flask on the left? The evidence suggests that iodine molecules constantly change from the solid phase to the gas phase according to the forward process, and that gaseous iodine molecules convert back to the solid phase according to the reverse process.

### Equilibrium Expressions and Constants

You have learned that some chemical systems have little tendency to react and others go readily to completion. In between these two extremes are the majority of reactions that reach a state of equilibrium with varying amounts of reactants unconsumed. If the reactants are not consumed, then not all the product predicted by the balanced chemical equation will be produced. According to the equation for the ammonia-producing reaction, two moles of ammonia should be produced when one mole of nitrogen and three moles of hydrogen react. Because the reaction reaches a state of equilibrium, however, fewer than two moles of ammonia will actually be obtained. Chemists need to be able to predict the yield of a reaction.

In 1864, the Norwegian chemists Cato Maximilian Guldberg and Peter Waage proposed the law of chemical equilibrium, which states that at a given temperature, a chemical system may reach a state in which a particular ratio of reactant and product concentrations has a constant value. For example, the general equation for a reaction at equilibrium can be written as follows.

\[ a\text{A} + b\text{B} \rightleftharpoons c\text{C} + d\text{D} \]

A and B are the reactants; C and D the products. The coefficients in the balanced equation are \(a\), \(b\), \(c\), and \(d\). If the law of chemical equilibrium is applied to this reaction, the following ratio is obtained.

\[ K_{\text{eq}} = \frac{[\text{C}]^c[\text{D}]^d}{[\text{A}]^a[\text{B}]^b} \]

This ratio is called the equilibrium constant expression. The square brackets indicate the molar concentrations of the reactants and products at equilibrium in mol/L. The equilibrium constant, \( K_{\text{eq}} \), is the numerical value of the ratio of product concentrations to reactant concentrations, with each concentration raised to the power corresponding to its coefficient in the balanced equation. The value of \( K_{\text{eq}} \) is constant only at a specified temperature.

How can you interpret the size of the equilibrium constant? Recall that ratios, or fractions, with large numerators are larger numbers than fractions...
with large denominators. For example, compare the ratio 5/1 with 1/5. Five is a larger number than one-fifth. Because the product concentrations are in the numerator of the equilibrium expression, a numerically large \( K_{eq} \) means that the equilibrium mixture contains more products than reactants. Similarly, a numerically small \( K_{eq} \) means that the equilibrium mixture contains more reactants than products.

\[
K_{eq} > 1: \text{More products than reactants at equilibrium.}
\]

\[
K_{eq} < 1: \text{More reactants than products at equilibrium.}
\]

**Constants for homogeneous equilibria** How would you write the equilibrium constant expression for this reaction in which hydrogen and iodine react to form hydrogen iodide?

\[
\text{H}_2(g) + \text{I}_2(g) \rightleftharpoons 2\text{HI}(g)
\]

This reaction is a **homogeneous equilibrium**, which means that all the reactants and products are in the same physical state. All participants are gases. To begin writing the equilibrium constant expression, place the product concentration in the numerator and the reactant concentrations in the denominator.

\[
\frac{[\text{HI}]}{[\text{H}_2][\text{I}_2]}
\]

The expression becomes equal to \( K_{eq} \) when you add the coefficients from the balanced chemical equation as exponents.

\[
K_{eq} = \frac{[\text{HI}]^2}{[\text{H}_2][\text{I}_2]}
\]

\( K_{eq} \) for this homogeneous equilibrium at 731 K is 49.7. Note that 49.7 has no units. In writing equilibrium constant expressions, it’s customary to omit units. Considering the size of \( K_{eq} \), are there more products than reactants present at equilibrium?

**EXAMPLE PROBLEM 18-1**

**Equilibrium Constant Expressions for Homogeneous Equilibria**

Write the equilibrium constant expression for the reaction in which ammonia gas is produced from hydrogen and nitrogen.

\[
\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)
\]

**Analyze the Problem**

You have been given the equation for the reaction, which provides the information needed to write the equilibrium constant expression. The equilibrium is homogeneous because the reactants and product are in the same physical state. The form of the equilibrium constant expression is \( K_{eq} = \frac{[\text{C}]^m[\text{B}]^n}{[\text{A}]^p} \).

**Known**

\[
[C] = [\text{NH}_3] \quad \text{coefficient NH}_3 = 2
\]

\[
[A] = [\text{N}_2] \quad \text{coefficient N}_2 = 1
\]

\[
[B] = [\text{H}_2] \quad \text{coefficient H}_2 = 3
\]

**Unknown**

\( K_{eq} = ? \)
Equilibrium: A State of Dynamic Balance

You have learned to write $K_{eq}$ expressions for homogeneous equilibria, those in which all reactants and products are in the same physical state. When the reactants and products of a reaction are present in more than one physical state, the equilibrium is called a heterogeneous equilibrium.

When ethanol is placed in a closed flask, a liquid-vapor equilibrium is established, as illustrated in Figure 18-5.

\[ C_2H_5OH(l) \rightleftharpoons C_2H_5OH(g) \]

To write the equilibrium constant expression for this process, you would form a ratio of the product to the reactant. At a given temperature, the ratio would have a constant value $K$.

\[ K = \frac{[C_2H_5OH(g)]}{[C_2H_5OH(l)]} \]

Note that the term in the denominator is the concentration of liquid ethanol. Because liquid ethanol is a pure substance, its concentration is constant at a given temperature. That’s because the concentration of a pure substance is its density in moles per liter. At any given temperature, density does not change. No matter how much or how little $C_2H_5OH$ is present, its concentration remains constant. Therefore, the term in the denominator is a constant and can be combined with $K$.

\[ K[C_2H_5OH(l)] = [C_2H_5OH(g)] = K_{eq} \]

The equilibrium constant expression for this phase change is

\[ K_{eq} = [C_2H_5OH(g)] \]
Chapter 18 Chemical Equilibrium

EXAMPLE PROBLEM 18-2

Equilibrium Constant Expressions for Heterogeneous Equilibria

Write the equilibrium constant expression for the decomposition of baking soda (sodium hydrogen carbonate).

\[ 2\text{NaHCO}_3(s) \rightleftharpoons \text{Na}_2\text{CO}_3(s) + \text{CO}_2(g) + \text{H}_2\text{O}(g) \]

1. Analyze the Problem

You are given a heterogeneous equilibrium involving gases and solids. The general form of the equilibrium constant expression for this reaction is

\[ K_{eq} = \frac{[C][D][E]^m}{[A]^n[B]^p}. \]

Because the reactant and one of the products are solids with constant concentrations, they can be omitted from the equilibrium constant expression.

Known

\[
\begin{align*}
[C] &= [\text{Na}_2\text{CO}_3] \\
[D] &= [\text{CO}_2] \\
[E] &= [\text{H}_2\text{O}] \\
[A] &= [\text{NaHCO}_3]
\end{align*}
\]

coefficient \( \text{Na}_2\text{CO}_3 = 1 \)
coefficient \( \text{CO}_2 = 1 \)
coefficient \( \text{H}_2\text{O} = 1 \)
coefficient \( \text{NaHCO}_3 = 2 \)

Unknown

equilibrium constant expression = ?

2. Solve for the Unknown

Write a ratio with the concentrations of the products in the numerator and the concentration of the reactant in the denominator.

\[
\frac{[\text{Na}_2\text{CO}_3][\text{CO}_2][\text{H}_2\text{O}]}{[\text{NaHCO}_3]}
\]

Leave out \( [\text{NaHCO}_3] \) and \( [\text{Na}_2\text{CO}_3] \) because they are solids.

\[
[\text{CO}_2][\text{H}_2\text{O}]
\]

Because the coefficients of \( [\text{CO}_2] \) and \( [\text{H}_2\text{O}] \) are 1, the expression is complete.

\[ K_{eq} = [\text{CO}_2][\text{H}_2\text{O}] \]

3. Evaluate the Answer

The expression correctly applies the law of chemical equilibrium to the equation.
Determined Value of Equilibrium Constants

For a given reaction at a given temperature, $K_{eq}$ will always be the same regardless of the initial concentrations of reactants and products. To test the truth of this statement, three experiments were carried out to investigate this reaction:

$$H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$$

The results are summarized in Table 18-1. In trial 1, 1.0000 mol $H_2$ and 2.0000 mol $I_2$ are placed in a 1.0000-L vessel. These initial concentrations have the symbols $[H_2]_0$ and $[I_2]_0$. No HI is present at the beginning of trial 1. In trial 2, only HI is present at the start of the experiment. In trial 3, each of the three participants has the same initial concentration.

When equilibrium is established, the concentration of each substance is determined experimentally. In Table 18-1, the symbol $[HI]_{eq}$ represents the concentration of HI at equilibrium. Note that the equilibrium concentrations are not the same in the three trials, yet when each set of equilibrium concentrations is put into the equilibrium constant expression, the value of $K_{eq}$ is the same. Each set of equilibrium concentrations represents an equilibrium position. Although an equilibrium system has only one value for $K_{eq}$ at a particular temperature, it has an unlimited number of equilibrium positions. Equilibrium positions depend upon the initial concentrations of the reactants and products.

The large value of $K_{eq}$ for the reaction $H_2(g) + I_2(g) \rightleftharpoons 2HI(g)$ means that at equilibrium the product is present in larger amount than the reactants. Many equilibria, however, have small $K_{eq}$ values. Do you remember what this means? $K_{eq}$ for the equilibrium $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$ equals $4.6 \times 10^{-31}$ at 298 K. A $K_{eq}$ this small means that the product, NO, is practically nonexistent at equilibrium.

Table 18-1

<table>
<thead>
<tr>
<th>Trial</th>
<th>$[H_2]_0$ (M)</th>
<th>$[I_2]_0$ (M)</th>
<th>$[HI]_0$ (M)</th>
<th>$[H_2]_{eq}$ (M)</th>
<th>$[I_2]_{eq}$ (M)</th>
<th>$[HI]_{eq}$ (M)</th>
<th>$K_{eq} = \frac{[HI]^2}{[H_2][I_2]}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>2.0000</td>
<td>0.0</td>
<td>0.06587</td>
<td>1.0659</td>
<td>1.8682</td>
<td>$\frac{[1.8682]^2}{[0.06587][1.0659]} = 49.70$</td>
</tr>
<tr>
<td>2</td>
<td>0.0</td>
<td>0.0</td>
<td>5.0000</td>
<td>0.5525</td>
<td>0.5525</td>
<td>3.8950</td>
<td>$\frac{[3.8950]^2}{[0.5525][0.5525]} = 49.70$</td>
</tr>
<tr>
<td>3</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>0.2485</td>
<td>0.2485</td>
<td>1.7515</td>
<td>$\frac{[1.7515]^2}{[0.2485][0.2485]} = 49.70$</td>
</tr>
</tbody>
</table>
EXAMPLE PROBLEM 18-3

Calculating the Value of Equilibrium Constants

Calculate the value of $K_{eq}$ for the equilibrium constant expression

$$K_{eq} = \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3}$$

given concentration data at one equilibrium position:

$$[\text{NH}_3] = 0.933 \text{ mol/L}, \ [\text{N}_2] = 0.533 \text{ mol/L}, \ [\text{H}_2] = 1.600 \text{ mol/L}.$$

1. **Analyze the Problem**

You have been given the equilibrium constant expression and the concentration of each reactant and product. You must calculate the equilibrium constant. Because the reactant, $\text{H}_2$, has the largest concentration and is raised to the third power in the denominator, $K_{eq}$ is likely to be less than 1.

### Known Unknown

- **Known:** $K_{eq} = \frac{[\text{NH}_3]^2}{[\text{N}_2][\text{H}_2]^3}$
- **Unknown:** $K_{eq} = ?$

- $[\text{NH}_3] = 0.933 \text{ mol/L}$
- $[\text{N}_2] = 0.533 \text{ mol/L}$
- $[\text{H}_2] = 1.600 \text{ mol/L}$

2. **Solve for the Unknown**

Substitute the known values into the equilibrium constant expression and calculate its value.

$$K_{eq} = \frac{[0.933]^2}{[0.533][1.600]^3} = 0.399$$

3. **Evaluate the Answer**

The smallest number of significant figures in the given data is three. Therefore, the answer is correctly stated with three digits. The calculation is correct and, as predicted, the value of $K_{eq}$ is less than 1.

PRACTICE PROBLEMS

3. Calculate $K_{eq}$ for the equilibrium in Practice Problem 1a on page 565 using the data $[\text{N}_2\text{O}_4] = 0.0185 \text{ mol/L}$ and $[\text{NO}_2] = 0.0627 \text{ mol/L}$.

4. Calculate $K_{eq}$ for the equilibrium in Practice Problem 1b on page 565 using the data $[\text{CO}] = 0.0613 \text{ mol/L}$, $[\text{H}_2] = 0.1839 \text{ mol/L}$, $[\text{CH}_4] = 0.0387 \text{ mol/L}$, and $[\text{H}_2\text{O}] = 0.0387 \text{ mol/L}$.

Section 18.1 Assessment

5. How does the concept of reversibility explain the establishment of equilibrium?

6. What characteristics define a system at equilibrium?

7. When you write an equilibrium constant expression, how do you decide what goes in the numerator and in the denominator?

8. **Thinking Critically** Determine whether the following statement is correct: When chemical equilibrium exists, the reactant and product concentrations remain constant and the forward and reverse reactions cease. Explain your answer.

9. **Using Numbers** Determine the value of $K_{eq}$ at 400 K for the decomposition of phosphorus pentachloride if $[\text{PCl}_5] = 0.135 \text{ mol/L}$, $[\text{PCl}_3] = 0.550 \text{ mol/L}$, and $[\text{Cl}_2] = 0.550 \text{ mol/L}$. The equation for the reaction is: $\text{PCl}_5(g) \rightleftharpoons \text{PCl}_3(g) + \text{Cl}_2(g)$
Manufacturers are aware that it makes good sense to minimize waste by finding ways of using leftover materials. Manufacturing new products from byproducts adds to profits and eliminates the problem of disposing of waste without causing environmental damage. What part does equilibrium play in promoting cost-cutting efficiency?

**Le Châtelier’s Principle**

Suppose that the byproducts of an industrial process are the gases carbon monoxide and hydrogen and a company chemist believes these gases can be combined to produce the fuel methane (CH₄) using this reaction.

\[
\text{CO(g)} + 3\text{H}_2(g) \rightarrow \text{CH}_4(g) + \text{H}_2\text{O(g)} \quad \Delta H^\circ = -206.5 \text{ kJ}
\]

When the industrial chemist places CO and H₂ in a closed reaction vessel at 1200 K, the reaction establishes this equilibrium position (equilibrium position 1).

\[
\begin{align*}
\text{CO(g)} & \quad 0.30000 M \\
3\text{H}_2(g) & \quad 0.10000 M \\
\text{CH}_4(g) & \quad 0.05900 M \\
\text{H}_2\text{O(g)} & \quad 0.02000 M 
\end{align*}
\]

Inserting these concentrations into the equilibrium expression gives an equilibrium constant equal to 3.933.

\[
K_{eq} = \frac{[\text{CH}_4][\text{H}_2\text{O}]}{[\text{CO}][\text{H}_2]^3} = \frac{(0.05900)(0.02000)}{(0.30000)(0.10000)^3} = 3.933
\]

Unfortunately, a methane concentration of 0.05900 mol/L in the equilibrium mixture is too low to be of any practical use. Can the chemist change the equilibrium position and thereby increase the amount of methane?

In 1888, the French chemist Henri-Louis Le Châtelier discovered that there are ways to control equilibria to make reactions, including this one, more productive. He proposed what is now called **Le Châtelier’s principle**: If a stress is applied to a system at equilibrium, the system shifts in the direction that relieves the stress. A stress is any kind of change in a system at equilibrium that upsets the equilibrium. You can use Le Châtelier’s principle to predict how changes in concentration, volume (pressure), and temperature affect equilibrium. Changes in volume and pressure are interrelated because decreasing the volume of a reaction vessel at constant temperature increases the pressure inside. Conversely, increasing the volume decreases the pressure.

**Changes in concentration** What happens if the industrial chemist injects additional carbon monoxide into the reaction vessel, raising the concentration of carbon monoxide from 0.30000 M to 1.00000 M? The higher carbon monoxide concentration immediately increases the number of effective collisions between CO and H₂ molecules and unbalances the equilibrium. The rate of the forward reaction increases, as indicated by the longer arrow to the right.

\[
\text{CO(g)} + 3\text{H}_2(g) \rightleftharpoons \text{CH}_4(g) + \text{H}_2\text{O(g)}
\]
In time, the rate of the forward reaction slows down as the concentrations of CO and $H_2$ decrease. Simultaneously, the rate of the reverse reaction increases as more and more $CH_4$ and $H_2O$ molecules are produced. Eventually, a new equilibrium position (position 2) is established with these concentrations.

$$\text{CO(g)} + 3\text{H}_2(g) \rightleftharpoons \text{CH}_4(g) + \text{H}_2\text{O(g)}$$

$$0.99254M \quad 0.07762M \quad 0.06648M \quad 0.02746M$$

$$K_{eq} = \frac{[\text{CH}_4][\text{H}_2\text{O}]}{[\text{CO}][\text{H}_2]^3} = \frac{(0.06648)(0.02746)}{(0.99254)(0.07762)^3} = 3.933$$

Note that although $K_{eq}$ has not changed, the new equilibrium position results in the desired effect—an increased concentration of methane. The results of this experiment are summarized in Table 18-2.

### Table 18-2

<table>
<thead>
<tr>
<th>Equilibrium position</th>
<th>$[\text{CO}]_{eq}$ M</th>
<th>$[\text{H}<em>2]</em>{eq}$ M</th>
<th>$[\text{CH}<em>4]</em>{eq}$ M</th>
<th>$[\text{H}<em>2\text{O}]</em>{eq}$ M</th>
<th>$K_{eq}$ M</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.30000</td>
<td>0.10000</td>
<td>0.05900</td>
<td>0.02000</td>
<td>3.933</td>
</tr>
<tr>
<td>2</td>
<td>0.99254</td>
<td>0.07762</td>
<td>0.06648</td>
<td>0.02746</td>
<td>3.933</td>
</tr>
</tbody>
</table>

Could you have predicted this result using Le Châtelier’s principle? Yes. Think of the increased concentration of CO as a stress on the equilibrium. The equilibrium system reacts to the stress by consuming CO at an increased rate. This response, called a shift to the right, forms more $CH_4$ and $H_2O$. Any increase in the concentration of a reactant results in a shift to the right and additional product.

Suppose that rather than injecting more reactant, the chemist decides to remove a product ($H_2O$) by adding a desiccant to the reaction vessel. Recall from Chapter 11 that a desiccant is a substance that absorbs water. What does Le Châtelier’s principle predict the equilibrium will do in response to a decrease in the concentration of water (the stress)? You are correct if you said that the equilibrium shifts in the direction that will tend to bring the concentration of water back up. That is, the equilibrium shifts to the right and results in additional product.

It might be helpful to think about the produce vendor in Figure 18-6 who wants to keep a display of vegetables looking neat and tempting. The vendor must constantly add new vegetables to fill the empty spaces created when customers buy the vegetables. Similarly, the equilibrium reaction attempts to restore the lost water by producing more water. As a result, more water and more methane are produced. In any equilibrium, the removal of a product results in a shift to the right and the production of more product.

The equilibrium position also can be shifted to the left, toward the reactants. Do you have an idea how? Le Châtelier’s principle predicts that if additional product is added to a reaction at equilibrium, the reaction will shift to the left. The stress is relieved by converting products to reactants. If one of the reactants is removed, a similar shift to the left will occur.

When predicting the results of a stress on an equilibrium using Le Châtelier’s principle, it is important to have the equation for the reaction in...
view. The effects of changing concentration are summarized in Figure 18-7. Refer to this figure as you practice applying Le Châtelier’s principle.

**Changes in volume** Consider again the reaction for making methane from byproduct gases.

\[
CO(g) + 3H_2(g) \rightleftharpoons CH_4(g) + H_2O(g)
\]

Can this reaction be forced to produce more methane by changing the volume of the reaction vessel? Suppose the vessel’s volume can be changed using a piston-like device similar to the one shown in Figure 18-8. If the piston is forced downward, the volume of the system decreases. You have learned that Boyle’s law says that decreasing the volume at constant temperature increases the pressure. The increased pressure is a stress on the reaction at equilibrium. How does the equilibrium respond to the disturbance and relieve the stress?

Recall that the pressure exerted by an ideal gas depends upon the number of gas particles that collide with the walls of the vessel. The more gas particles contained in the vessel, the greater the pressure. If the number of gas particles is increased at constant temperature, the pressure of the gas increases. Similarly, if the number of gas particles is decreased, the pressure also decreases. How does this relationship between numbers of gas particles and pressure apply to the reaction for making methane? Compare the number of moles of gaseous reactants in the equation with the number of moles of gaseous products. For every two moles of gaseous products (1 mol CH₄ and 1 mol H₂O), four moles of gaseous reactants are consumed (1 mol CO and 3 mol H₂), a net decrease of two moles. If you apply Le Châtelier’s principle, you can see that the equilibrium can relieve the stress of increased pressure by shifting to the right. This shift decreases the total number of moles of gas and thus, the pressure inside the reaction vessel decreases. Although the shift to the right does not reduce the pressure to its original value, it has the desired effect—the equilibrium produces more methane. See Figure 18-8.

**Figure 18-7**

The addition or removal of a reactant or product shifts the equilibrium in the direction that relieves the stress. Note the unequal arrows, which indicate the direction of the shift. How would the reaction shift if you added H₂? Removed CH₄?

**Figure 18-8**

In a, the reaction between CO and H₂ is at equilibrium. In b, the piston has been lowered decreasing the volume and increasing the pressure. The outcome is seen in c. More molecules of the products have formed. Their formation helped relieve the stress on the system. How do the numbers of particles in a and c compare?
Changing the volume (and pressure) of an equilibrium system shifts the equilibrium only if the number of moles of gaseous reactants is different from the number of moles of gaseous products. Changing the volume of a reaction vessel containing the equilibrium $\text{H}_2(g) + \text{I}_2(g) \rightleftharpoons 2\text{HI}(g)$ would cause no shift in the equilibrium because the number of moles of gas is the same on both sides of the equation.

**Changes in temperature** You have now learned that changes in concentration and volume change the equilibrium position by causing shifts to the right or left, but they do not change the equilibrium constant. A change in temperature, however, alters both the equilibrium position and the equilibrium constant. To understand how a change in temperature affects an equilibrium, recall that virtually every chemical reaction is either endothermic or exothermic. For example, the reaction for making methane has a negative $\Delta H^\circ$, which means that the forward reaction is exothermic and the reverse reaction is endothermic.

$$\text{CO}(g) + 3\text{H}_2(g) \rightleftharpoons \text{CH}_4(g) + \text{H}_2\text{O}(g) \quad \Delta H^\circ = -206.5 \text{ kJ}$$

Heat can be thought of as a product in the forward reaction and a reactant in the reverse reaction. You can see this by reading the equation forward and backward.

$$\text{CO}(g) + 3\text{H}_2(g) \rightleftharpoons \text{CH}_4(g) + \text{H}_2\text{O}(g) + \text{heat}$$

How could an industrial chemist regulate the temperature to increase the amount of methane in the equilibrium mixture? According to Le Châtelier’s principle, if heat is added, the reaction shifts in the direction in which heat is used up; that is, the reaction shifts to the left. A shift to the left means a decrease in the concentration of methane because methane is a reactant in the reverse reaction. However, lowering the temperature shifts the equilibrium to the right because the forward reaction liberates heat and relieves the stress. In shifting to the right, the equilibrium produces more methane.

Any change in temperature results in a change in $K_{eq}$. Recall that the larger the value of $K_{eq}$, the more product is found in the equilibrium mixture. Thus, for the methane-producing reaction, $K_{eq}$ increases in value when the temperature is lowered and decreases when the temperature is raised.

**Figure 18-9** shows how another equilibrium responds to changes in temperature. The endothermic equilibrium is described by this equation.

$$\text{heat} + \text{Co(H}_2\text{O)}_6^{2+}(aq) + 4\text{Cl}^-(aq) \rightleftharpoons \text{CoCl}_4^{2-}(aq) + 6\text{H}_2\text{O}(l)$$

pink blue

---

**Figure 18-9**

At room temperature, the solution in **a** is an equilibrium mixture of pink reactants and blue products. When the temperature is lowered in **b**, this endothermic reaction shifts toward the pink reactants. In **c**, the stress of higher temperature causes the reaction to shift toward the blue products.
For the exothermic reaction between CO and H₂, raising the temperature shifts the equilibrium to the left (top equation). Lowering the temperature results in a shift to the right (second equation). The opposite is true for the endothermic reaction involving cobalt and chloride ions (third and last equations).

At room temperature, this mixture of aqueous ions appears violet because it contains significant amounts of pink \(\text{Co(H}_2\text{O)}_{6}^{2+}\text{(aq)}\) and blue \(\text{CoCl}_4^{2-}\text{(aq)}\). When cooled in an ice bath, the equilibrium mixture turns pink. The removal of heat is a stress on the equilibrium. The stress is relieved by an equilibrium shift to the left, generating more heat and producing more pink \(\text{Co(H}_2\text{O)}_{6}^{2+}\text{(aq)}\) ions. When heat is added, the equilibrium mixture appears blue because the equilibrium shifts to the right to absorb the additional heat. As a result, more blue \(\text{CoCl}_4^{2-}\) ions are created. You can investigate this equilibrium system further in the miniLAB below. The diagram in Figure 18-10 shows the effect of heating and cooling on exothermic and endothermic reactions.

Changes in concentration, volume, and temperature make a difference in the amount of product formed in a reaction. Can a catalyst also affect product concentration? A catalyst speeds up a reaction, but it does so equally in both directions. Therefore, a catalyzed reaction reaches equilibrium more quickly, but with no change in the amount of product formed. Read the Chemistry and Technology feature at the end of this chapter to learn how Le Châtelier’s principle is applied to an important industrial process.

### miniLAB

**Shifts in Equilibrium**

**Observing and Inferring** Le Châtelier’s principle states that if a stress is placed on a reaction at equilibrium, the system will shift in a way that will relieve the stress. In this experiment, you will witness an equilibrium shift in a colorful way.

**Materials** test tubes (2); 10-mL graduated cylinder; 250-mL beaker; concentrated hydrochloric acid; 0.1\(M\) \(\text{CoCl}_2\) solution; ice bath; table salt; hot plate

**Procedures**

1. Place about 2 mL of 0.1\(M\) \(\text{CoCl}_2\) solution in a test tube. Record the color of the solution.
2. Add about 3 mL of concentrated HCl to the test tube. Record the color of the solution. **CAUTION:** HCl can burn skin and clothing.
3. Add enough water to the test tube to make a color change occur. Record the color.
4. Add about 2 mL of 0.1\(M\) \(\text{CoCl}_2\) to another test tube. Add concentrated HCl dropwise until the solution turns purple. If the solution becomes blue, add water until it turns purple.
5. Place the test tube in an ice bath that has had some salt sprinkled into the ice water. Record the color of the solution in the test tube.
6. Place the test tube in a hot water bath that is at least 70°C. Record the color of the solution.

**Analysis**

1. The equation for the reversible reaction in this experiment is \(\text{Co(H}_2\text{O)}_{6}^{2+} + 4\text{Cl}^- \rightleftharpoons \text{CoCl}_4^{2-} + 6\text{H}_2\text{O}\)
   - pink
   - blue

   Use the equation to explain your observations of color in steps 1–3.
2. Explain how the equilibrium shifts when energy is added or removed.
A biological equilibrium If you have ever traveled to the mountains for a strenuous activity such as skiing, hiking, or mountain climbing, it is likely that you have felt tired and lightheaded for a time. This feeling is a result of the fact that at high altitudes, the air is thinner and contains fewer oxygen molecules. An important equilibrium in your body is disturbed. That equilibrium is represented by the following equation.

\[ \text{Hgb(aq)} + \text{O}_2(g) \rightleftharpoons \text{Hgb(O}_2\text{)(aq)} \]

Hgb and Hgb(O₂) are greatly simplified formulas for hemoglobin and oxygenated hemoglobin, respectively. Hemoglobin is the blood protein that transports oxygen from your lungs to your muscles and other tissues where it is used in the metabolic processes that produce energy. What happens to this equilibrium at an altitude where both the atmospheric pressure and oxygen concentration are lower than normal for your body? Applying Le Châtelier’s principle, the equilibrium shifts to the left to produce more oxygen. This shift reduces the amount of Hgb(O₂) in your blood and, therefore, the supply of oxygen to your muscles and other tissues.

After spending some time in the mountains, you probably noticed that your fatigue lessened. That’s because your body adapted to the reduced oxygen concentration by producing more Hgb, which shifts the equilibrium back to the right and increases the amount of Hgb(O₂) in your blood. The Sherpas in Figure 18-11, who live and work in the mountains, do not experience discomfort because the equilibrium systems in their blood are adapted to high-altitude conditions.

Figure 18-11
The equilibrium in the blood of Sherpas is adjusted to the lower level of oxygen in the air at high altitudes. Do you think Sherpas would experience a period of adjustment if they moved to sea level?
When a reaction has a large $K_{eq}$, the products are favored at equilibrium. That means that the equilibrium mixture contains more products than reactants. Conversely, when a reaction has a small $K_{eq}$, the reactants are favored at equilibrium, which means that the equilibrium mixture contains more reactants than products. Knowing the size of the equilibrium constant can help a chemist decide whether a reaction is practical for making a particular product.

Calculating Equilibrium Concentrations

The equilibrium constant expression can be useful in another way. Knowing the equilibrium constant expression, a chemist can calculate the equilibrium concentration of any substance involved in a reaction if the concentrations of all other reactants and products are known.

Suppose the industrial chemist that you read about earlier knows that at 1200 K, $K_{eq}$ equals 3.933 for the reaction that forms methane from $H_2$ and CO. How much methane would actually be produced? If the concentrations of $H_2$, CO, and $H_2O$ are known, the concentration of $CH_4$ can be calculated.

\[
\text{CO(g)} + 3\text{H}_2(\text{g}) \rightleftharpoons \text{CH}_4(\text{g}) + \text{H}_2\text{O(\text{g})}
\]

\[
0.850M \quad 1.333M \quad ?M \quad 0.286M
\]

The first thing the chemist would do is write the equilibrium constant expression.

\[
K_{eq} = \frac{[\text{CH}_4][\text{H}_2\text{O}]}{[\text{CO}][\text{H}_2]^3}\n\]

The equation can be solved for the unknown $[\text{CH}_4]$ by multiplying both sides of the equation by $[\text{CO}][\text{H}_2]^3$ and dividing both sides by $[\text{H}_2\text{O}]$.

\[
[\text{CH}_4] = K_{eq} \times \frac{[\text{CO}][\text{H}_2]^3}{[\text{H}_2\text{O}]}\n\]

All the known concentrations and the value of $K_{eq}$ (3.933) can now be substituted into the equilibrium constant expression.

\[
[\text{CH}_4] = 3.933 \times \frac{(0.850)(1.333)^3}{(0.286)} = 27.7 \text{ mol/L}\n\]

The equilibrium concentration of $\text{CH}_4$ is 27.7 mol/L.

At this point an industrial chemist would evaluate whether an equilibrium concentration of 27.7 mol/L was sufficient to make the conversion of waste CO and $H_2$ to methane practical. Methane is becoming the fuel of choice for heating homes and cooking food. It is also the raw material for the manufacture of many products including acetylene and formic acid. Increasingly, it is being used as the energy source of fuel cells. At present, methane is relatively inexpensive but as demand grows, the cost will increase. Can the cost of a process which produces 27.7 mol/L $\text{CH}_4$ compete with the cost of obtaining methane from underground deposits? This is an important question for the manufacturer. Figure 18-12 shows a surprising source of methane in the atmosphere.
EXAMPLE PROBLEM 18-4

Calculating Equilibrium Concentrations

At 1405 K, hydrogen sulfide, also called rotten egg gas because of its bad odor, decomposes to form hydrogen and a diatomic sulfur molecule, \( S_2 \). The equilibrium constant for the reaction is \( 2.27 \times 10^{-3} \).

\[
2H_2S(g) \rightleftharpoons 2H_2(g) + S_2(g)
\]

What is the concentration of hydrogen gas if \([S_2]\) = 0.0540 mol/L and \([H_2S]\) = 0.184 mol/L?

1. **Analyze the Problem**
   
   You have been given \( K_{eq} \) and two of the three variables in the equilibrium constant expression. The equilibrium expression can be solved for \([H_2]\). \( K_{eq} \) is less than one, so more reactants than products are in the equilibrium mixture. Thus, you can predict that \([H_2]\) will be less than 0.184 mol/L, the concentration of the reactant \( H_2S \).

   **Known**
   
   \[ K_{eq} = 2.27 \times 10^{-3} \]
   \[ [S_2] = 0.0540 \text{ mol/L} \]
   \[ [H_2S] = 0.184 \text{ mol/L} \]

   **Unknown**
   
   \([H_2]\)  \( \text{mol/L} \)

2. **Solve for the Unknown**

   Write the equilibrium constant expression.

   \[
   \frac{[H_2]^2[S_2]}{[H_2S]^2} = K_{eq}
   \]

   Solve the equation for \([H_2]^2\) by dividing both sides of the equation by \([S_2]\) and multiplying both sides by \([H_2S]^2\).

   \[
   [H_2]^2 = K_{eq} \times \frac{[H_2S]^2}{[S_2]}
   \]

   Substitute the known quantities into the expression and solve for \([H_2]\).

   \[
   [H_2]^2 = 2.27 \times 10^{-3} \times \frac{(0.184)^2}{(0.054)} = 1.42 \times 10^{-3}
   \]

   \[
   [H_2] = \sqrt{1.42 \times 10^{-3}} = 0.0377 \text{ mol/L}
   \]

   The equilibrium concentration of \( H_2 \) is 0.0377 mol/L.

3. **Evaluate the Answer**

   All of the data for the problem have three significant figures, so the answer is correctly stated with three digits. As predicted, the equilibrium concentration of \( H_2 \) is less than 0.184 mol/L.

**PRACTICE PROBLEMS**

16. At a certain temperature, \( K_{eq} = 10.5 \) for the equilibrium

   \[
   CO(g) + 2H_2(g) \rightleftharpoons CH_3OH(g)
   \]

   Calculate these concentrations:

   **a.** \([CO]\) in an equilibrium mixture containing 0.933 mol/L \( H_2 \) and 1.32 mol/L \( CH_3OH \)

   **b.** \([H_2]\) in an equilibrium mixture containing 1.09 mol/L \( CO \) and 0.325 mol/L \( CH_3OH \)

   **c.** \([CH_3OH]\) in an equilibrium mixture containing 0.0661 mol/L \( H_2 \) and 3.85 mol/L \( CO \).
Solubility Equilibria

Some ionic compounds dissolve readily in water and some barely dissolve at all. Sodium chloride, or table salt, is typical of the soluble ionic compounds. On dissolving, all ionic compounds dissociate into ions.

\[ \text{NaCl}(s) \rightarrow \text{Na}^+(aq) + \text{Cl}^-(aq) \]

Approximately 36 g NaCl dissolves in 100 mL of water at 273 K. Without this high solubility, sodium chloride couldn’t flavor and preserve foods like the pickles shown in Figure 18-13. Sodium chloride’s vital role as an electrolyte in blood chemistry also depends upon its high solubility.

Although high solubility in water is often beneficial, low solubility also is important in many applications. For example, although barium ions are toxic to humans, patients are required to ingest barium sulfate prior to having an X ray of the digestive tract taken. X rays taken without barium sulfate in the digestive system are not well defined. Why can patients safely ingest barium sulfate?

In water solution, barium sulfate dissociates according to this equation.

\[ \text{BaSO}_4(s) \rightarrow \text{Ba}^{2+}(aq) + \text{SO}_4^{2-}(aq) \]

As soon as the first product ions form, the reverse reaction begins to re-form the reactants according to this equation.

\[ \text{BaSO}_4(s) \rightleftharpoons \text{Ba}^{2+}(aq) + \text{SO}_4^{2-}(aq) \]

With time, the rate of the reverse reaction becomes equal to the rate of the forward reaction and equilibrium is established.

\[ \text{BaSO}_4(s) \rightleftharpoons \text{Ba}^{2+}(aq) + \text{SO}_4^{2-}(aq) \]

For sparingly soluble compounds, such as BaSO_4, the rates become equal when the concentrations of the aqueous ions are exceedingly small. Nevertheless, the solution at equilibrium is a saturated solution.

The equilibrium constant expression for the dissolving of BaSO_4 is

\[ K_{eq} = \frac{[\text{Ba}^{2+}][\text{SO}_4^{2-}]}{[\text{BaSO}_4]} \]

In the equilibrium expression, [BaSO_4] is constant because barium sulfate is a solid. This constant value is combined with \( K_{eq} \) by multiplying both sides of the equation by [BaSO_4].
The product of $K_{eq}$ and the concentration of the undissolved solid creates a new constant called the solubility product constant, $K_{sp}$. The **solubility product constant** is an equilibrium constant for the dissolving of a sparingly soluble ionic compound in water. The solubility product constant expression is

$$K_{sp} = [Ba^{2+}][SO_4^{2-}] = 1.1 \times 10^{-10} \text{ at } 298 \text{ K}$$

The solubility product constant expression is the product of the concentrations of the ions each raised to the power equal to the coefficient of the ion in the chemical equation. The small value of $K_{sp}$ indicates that products are not favored at equilibrium. Thus, few barium ions are present at equilibrium ($1.0 \times 10^{-3} \text{ M}$) and a patient can safely ingest a barium sulfate solution to obtain a clear X-ray like the one shown in Figure 18-14.

Here is another example.

$$\text{Mg(OH)}_2(s) \rightleftharpoons \text{Mg}^{2+}(aq) + 2\text{OH}^-(aq)$$

$$K_{sp} = [\text{Mg}^{2+}][\text{OH}^-]^2$$

$K_{sp}$ depends only on the concentrations of the ions in the saturated solution. However, to establish an equilibrium system, some undissolved solid, no matter how small the amount, must be present in the equilibrium mixture.

The solubility product constants for some ionic compounds are listed in Table 18-3. Note that they are all small numbers. Solubility product constants are measured and recorded only for sparingly soluble compounds.

**Using solubility product constants** The solubility product constants in Table 18-3 have been determined through careful experiments. $K_{sp}$ values are important because they can be used to determine the solubility of a sparingly soluble compound. Recall that the solubility of a compound in water is the amount of the substance that will dissolve in a given volume of water.

### Table 18-3

<table>
<thead>
<tr>
<th>Compound</th>
<th>$K_{sp}$</th>
<th>Compound</th>
<th>$K_{sp}$</th>
<th>Compound</th>
<th>$K_{sp}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Carbonates</td>
<td></td>
<td>Halides</td>
<td></td>
<td>Hydroxides</td>
<td></td>
</tr>
<tr>
<td>BaCO$_3$</td>
<td>$2.6 \times 10^{-9}$</td>
<td>CaF$_2$</td>
<td>$3.5 \times 10^{-11}$</td>
<td>Al(OH)$_3$</td>
<td>$4.6 \times 10^{-13}$</td>
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<td>CaCO$_3$</td>
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<td>PbBr$_2$</td>
<td>$6.6 \times 10^{-6}$</td>
<td>Ca(OH)$_2$</td>
<td>$5.0 \times 10^{-6}$</td>
</tr>
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<td>$2.5 \times 10^{-10}$</td>
<td>PbCl$_2$</td>
<td>$1.7 \times 10^{-5}$</td>
<td>Cu(OH)$_2$</td>
<td>$2.2 \times 10^{-20}$</td>
</tr>
<tr>
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<td>PbF$_2$</td>
<td>$3.3 \times 10^{-8}$</td>
<td>Fe(OH)$_2$</td>
<td>$4.9 \times 10^{-17}$</td>
</tr>
<tr>
<td>MgCO$_3$</td>
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<td>PbI$_2$</td>
<td>$9.8 \times 10^{-9}$</td>
<td>Fe(OH)$_3$</td>
<td>$2.8 \times 10^{-39}$</td>
</tr>
<tr>
<td>Ag$_2$CO$_3$</td>
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<td>AgCl</td>
<td>$1.8 \times 10^{-10}$</td>
<td>Mg(OH)$_2$</td>
<td>$5.6 \times 10^{-12}$</td>
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<td>AgBr</td>
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<td>$3 \times 10^{-17}$</td>
</tr>
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<td>AgI</td>
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<td>Phosphates</td>
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<td>Sulfates</td>
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<td>AlPO$_4$</td>
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<td>BaSO$_4$</td>
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</tr>
<tr>
<td>PbCrO$_4$</td>
<td>$2.3 \times 10^{-13}$</td>
<td>Ca$_3$(PO$_4$)$_2$</td>
<td>$2.1 \times 10^{-33}$</td>
<td>CaSO$_4$</td>
<td>$4.9 \times 10^{-5}$</td>
</tr>
<tr>
<td>Ag$_2$CrO$_4$</td>
<td>$1.1 \times 10^{-12}$</td>
<td>Mg$_3$(PO$_4$)$_2$</td>
<td>$1.0 \times 10^{-24}$</td>
<td>PbSO$_4$</td>
<td>$2.5 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

**Figure 18-14**

The presence of barium ions in the gastrointestinal system made the sharp definition of this X-ray possible.
Suppose you wish to determine the solubility of silver iodide (AgI) in mol/L at 298 K. The equilibrium equation and solubility product constant expression are

\[ \text{AgI(s)} \rightleftharpoons \text{Ag}^+(aq) + \text{I}^-(aq) \]
\[ K_{sp} = [\text{Ag}^+][\text{I}^-] = 8.5 \times 10^{-17} \text{ at 298 K} \]

The first thing you should do is let the symbol \( s \) represent the solubility of AgI; that is, the number of moles of AgI that dissolves in a liter of solution. The equation indicates that for every mole of AgI that dissolves, an equal number of moles of Ag\(^+\) ions forms in solution. Therefore, [Ag\(^+\)] equals \( s \). Every Ag\(^+\) has an accompanying I\(^-\) ion, so [I\(^-\)] also equals \( s \). Substituting \( s \) for [Ag\(^+\)] and [I\(^-\)], the \( K_{sp} \) expression becomes

\[ [\text{Ag}^+][\text{I}^-] = (s)(s) = s^2 = 8.5 \times 10^{-17} \]
\[ s = \sqrt{8.5 \times 10^{-17}} = 9.2 \times 10^{-9} \text{ mol/L}. \]

The solubility of AgI is \( 9.2 \times 10^{-9} \) mol/L at 298 K.
EXAMPLE PROBLEM 18-6

Calculating Ion Concentration from $K_{sp}$

Magnesium hydroxide is a white solid that is processed from seawater. Determine the hydroxide ion concentration at 298 K in a saturated solution of Mg(OH)$_2$ if the $K_{sp}$ equals $5.6 \times 10^{-12}$.

1. Analyze the Problem

You have been given the $K_{sp}$ for Mg(OH)$_2$. The moles of Mg$^{2+}$ ions in solution equal the moles of Mg(OH)$_2$ that dissolved, but the moles of OH$^-$ ions in solution are two times the moles of Mg(OH)$_2$ that dissolved. You can use these relationships to write the solubility product constant expression in terms of one unknown. Because the equilibrium expression is a third power equation, you can predict that [OH$^-$] will be approximately the cube root of $10^{-12}$, or approximately $10^{-4}$.

Known Unknown

$K_{sp} = 5.6 \times 10^{-12}$ [OH$^-$] = ? mol/L

2. Solve for the Unknown

Write the equation for the solubility equilibrium and the $K_{sp}$ expression.

Mg(OH)$_2$(s) $\rightleftharpoons$ Mg$^{2+}$(aq) + 2OH$^-$ (aq)

$K_{sp} = [\text{Mg}^{2+}][\text{OH}^-]^2 = 5.6 \times 10^{-12}$

Let $x$ equal [Mg$^{2+}$]. Because there are two OH$^-$ ions for every Mg$^{2+}$ ion, [OH$^-$] = 2x. Substitute these terms into the $K_{sp}$ expression and solve for $x$.

$(x)(2x)^2 = 5.6 \times 10^{-12}$

$(x)(4)(x)^2 = 5.6 \times 10^{-12}$

$4x^3 = 5.6 \times 10^{-12}$

$x^3 = \frac{5.6 \times 10^{-12}}{4} = 1.4 \times 10^{-12}$

$x = [\text{Mg}^{2+}] = \sqrt{1.4 \times 10^{-12}} = 1.1 \times 10^{-4}$ mol/L

Multiply [Mg$^{2+}$] by 2 to obtain [OH$^-$].

[OH$^-$] = 2[Mg$^{2+}$] = 2(1.1 $\times 10^{-4}$ mol/L) = 2.2 $\times 10^{-4}$ mol/L

3. Evaluate the Answer

The given $K_{sp}$ has two significant figures, so the answer is correctly stated with two digits. As predicted, [OH$^-$] is about $10^{-4}$ mol/L.
Predicting precipitates  You have seen that solubility product constants are useful for finding the solubility of an ionic compound and for calculating the concentrations of ions in a saturated solution. You also can use $K_{sp}$ to predict whether a precipitate will form when two ionic solutions are mixed. For example, will a precipitate form when equal volumes of 0.10 M aqueous solutions of iron(III) chloride (FeCl₃) and potassium hexacyanoferrate(II) (K₄Fe(CN)₆) are poured together? A double-replacement reaction might occur according to this equation.

$$4\text{FeCl}_3 + 3\text{K}_4\text{Fe(CN)}_6 \rightarrow 12\text{KCl} + \text{Fe}_4(\text{Fe(CN)}_6)_3$$

A precipitate is likely to form only if either product, KCl or Fe₄(Fe(CN)₆)₃, has low solubility. You are probably aware that KCl is a soluble compound and would be unlikely to precipitate. However, the $K_{sp}$ for Fe₄(Fe(CN)₆)₃ is a very small number, $3.3 \times 10^{-11}$, which suggests that Fe₄(Fe(CN)₆)₃ might be expected to precipitate if the concentrations of its ions are large enough. Perhaps you are wondering how large is large enough.

Table 18-4 shows the concentrations of the ions of reactants and products in the original solutions (0.10M FeCl₃ and 0.10M K₄Fe(CN)₆) and in the mixture immediately after equal volumes of the two solutions were mixed. Note that [Cl⁻] is three times as large as [Fe³⁺] because the ratio of Cl⁻ to Fe³⁺ in FeCl₃ is 3:1. Also note that [K⁺] is four times as large as [Fe(CN)₆⁴⁻] because the ratio of K⁺ to Fe(CN)₆⁴⁻ in K₄Fe(CN)₆ is 4:1. In addition, note that the concentration of each ion in the mixture is one-half its original concentration. This is because when equal volumes of two solutions are mixed, the same number of ions are dissolved in twice as much solution, so the concentration is reduced by one-half.

You can now use the data in the table to make a trial to see if the concentrations of Fe³⁺ and Fe(CN)₆⁴⁻ in the mixed solution exceed the value of $K_{sp}$ when substituted into the solubility product constant expression.

$$K_{sp} = [\text{Fe}^{3+}]^4[\text{Fe(CN)}_6^{4-}]^3$$

But first, remember that you have not determined whether the solution is saturated. When you make this substitution, it will not necessarily give the solubility product constant. Instead, it provides a number called the ion product, $Q_{sp}$. $Q_{sp}$ is a trial value that can be compared with $K_{sp}$.
EXAMPLE PROBLEM 18-7

**Predicting a Precipitate**
Predict whether a precipitate of PbCl₂ will form if 100 mL of 0.0100 M NaCl is added to 100 mL of 0.0200 M Pb(NO₃)₂.

1. **Analyze the Problem**
   You have been given equal volumes of two solutions with known concentrations. The concentrations of the initial solutions allow you to calculate the concentrations of Pb²⁺ and Cl⁻ ions in the mixed solution. The initial concentrations, when multiplied together in the solubility product constant expression, give an ion product of the order of 10⁻⁶, so it is probable that after dilution, Qₛₚ will be less than Kₛₚ (1.7 × 10⁻⁵) and PbCl₂ will not precipitate.

   Known          Unknown
   100 mL 0.0100 M NaCl          Qₛₚ > Kₛₚ?
   100 mL 0.0200 M Pb(NO₃)₂
   Kₛₚ = 1.7 × 10⁻⁵

2. **Solve for the Unknown**
   Write the equation for the dissolving of PbCl₂ and the ion product expression, Qₛₚ.
   \[
   \text{PbCl}_2(s) \rightleftharpoons \text{Pb}^{2+}(aq) + 2\text{Cl}^-(aq)
   \]
   \[
   Qₛₚ = [\text{Pb}^{2+}][\text{Cl}^-]^2
   \]
   Divide the concentrations of Cl⁻ and Pb²⁺ in half because on mixing, the volume doubles.
   \[
   [\text{Cl}^-] = \frac{0.0100 M}{2} = 0.00500 M
   \]
   \[
   [\text{Pb}^{2+}] = \frac{0.0200 M}{2} = 0.0100 M
   \]
   Substitute these values into the ion product expression.
   \[
   Qₛₚ = (0.0100)(0.00500)^2 = 2.5 \times 10^{-7}
   \]
   Compare Qₛₚ with Kₛₚ.
   \[
   Qₛₚ (2.5 \times 10^{-7}) < Kₛₚ (1.7 \times 10^{-5})
   \]
   A precipitate will not form.
3. Evaluate the Answer

As predicted, $Q_{sp}$ is less than $K_{sp}$. The Pb$^{2+}$ and Cl$^-$ ions are not present in high enough concentrations in the mixed solution to cause precipitation to occur.

PRACTICE PROBLEMS

19. Use $K_{sp}$ values from Table 18-3 to predict whether a precipitate will form when equal volumes of the following aqueous solutions are mixed.

| (a) | 0.10 M Pb(NO$_3$)$_2$ and 0.030 M NaF |
| (b) | 0.25 M K$_2$SO$_4$ and 0.010 M AgNO$_3$ |
| (c) | 0.20 M MgCl$_2$ and 0.0025 M NaOH |

Common Ion Effect

The solubility of lead chromate (PbCrO$_4$) in water is $4.8 \times 10^{-7}$ mol/L at 298 K. That means you can dissolve $4.8 \times 10^{-7}$ mol PbCrO$_4$ in 1.00 L of pure water. However, you can’t dissolve $4.8 \times 10^{-7}$ mol PbCrO$_4$ in 1.00 L of 0.10 M aqueous potassium chromate (K$_2$CrO$_4$) solution at that temperature. Why is PbCrO$_4$ less soluble in an aqueous K$_2$CrO$_4$ solution than in pure water?

problem-solving LAB

How does fluoride prevent tooth decay?

Using Numbers During the last half century, tooth decay has decreased significantly because minute quantities of fluoride ion ($6 \times 10^{-5}$ M) are being added to most public drinking water and many people are using toothpastes containing sodium fluoride or tin(II) fluoride. Use what you know about the solubility of ionic compounds and reversible reactions to explore the role of the fluoride ion in maintaining cavity-free teeth.

Analysis

Tooth enamel, the hard, protective outer layer of the tooth, is 98% hydroxyapatite (Ca$_5$(PO$_4$)$_3$OH). Although quite insoluble ($K_{sp} = 6.8 \times 10^{-37}$ in water), demineralization, which is the dissolving of hydroxyapatite, does occur especially when the saliva contains acids. The reverse reaction, remineralization, also occurs. Remineralization is the re-depositing of tooth enamel. When hydroxyapatite is in solution with fluoride ions, a double-replacement reaction can occur. Fluoride ion replaces the hydroxide ion to form fluoroapatite (Ca$_5$(PO$_4$)$_3$F), $K_{sp} = 1 \times 10^{-60}$. Fluoroapatite remineralizes the tooth enamel, thus partially displacing hydroxyapatite. Because fluoroapatite is less soluble than hydroxyapatite, destructive demineralization is reduced.

Thinking Critically

1. Write the equation for the dissolving of hydroxyapatite and its equilibrium constant expression. How do the conditions in the mouth differ from those of a true equilibrium?
2. Write the equation for the double-replacement reaction that occurs between hydroxyapatite and sodium fluoride.
3. Calculate the solubility of hydroxyapatite and fluoroapatite in water. Compare the solubilities.
4. What is the ion product constant ($Q_{sp}$) for the reaction if 0.00050 M NaF is mixed with an equal volume of 0.000015 M Ca$_5$(PO$_4$)$_3$OH? Will a precipitate form (re-mineralization)?
The equation for the PbCrO₄ solubility equilibrium and the solubility product constant expression are

\[
\text{PbCrO}_4(s) \rightleftharpoons \text{Pb}^{2+}(aq) + \text{CrO}_4^{2-}(aq).
\]

\[
K_{sp} = [\text{Pb}^{2+}][\text{CrO}_4^{2-}] = 2.3 \times 10^{-13} \text{ (in water)}
\]

\[
[\text{Pb}^{2+}][\text{CrO}_4^{2-}] = 2.3 \times 10^{-13} \text{ (in 0.10M K}_2\text{CrO}_4\text{)}
\]

The equation for the PbCrO₄ solubility equilibrium and the solubility product constant expression are

\[
\text{PbCrO}_4(s) \rightleftharpoons \text{Pb}^{2+}(aq) + \text{CrO}_4^{2-}(aq).
\]

\[
K_{sp} = [\text{Pb}^{2+}][\text{CrO}_4^{2-}] = 2.3 \times 10^{-13}
\]

Recall that \( K_{sp} \) is a constant at any given temperature, so if the concentration of either \( \text{Pb}^{2+} \) or \( \text{CrO}_4^{2-} \) increases when the system is at equilibrium, the concentration of the other ion must decrease. The product of the concentrations of the two ions must always equal \( K_{sp} \). The K₂CrO₄ solution contains \( \text{CrO}_4^{2-} \) ions before any PbCrO₄ dissolves. In this example, the \( \text{CrO}_4^{2-} \) ion is called a common ion because it is part of both PbCrO₄ and K₂CrO₄. A common ion is an ion that is common to two or more ionic compounds. The lowering of the solubility of a substance by the presence of a common ion is called the common ion effect.

**The common ion effect and Le Châtelier’s principle** A saturated solution of lead chromate (PbCrO₄) is shown in Figure 18-17a. Note the solid yellow PbCrO₄ in the bottom of the test tube. The solution and solid are in equilibrium according to this equation.

\[
\text{PbCrO}_4(s) \rightleftharpoons \text{Pb}^{2+}(aq) + \text{CrO}_4^{2-}(aq)
\]

When a solution of Pb(NO₃)₂ is added to the saturated PbCrO₄ solution, more solid PbCrO₄ precipitates, as you can see in Figure 18-17b. The Pb²⁺ ion, common to both Pb(NO₃)₂ and PbCrO₄, reduces the solubility of PbCrO₄. Can this precipitation of PbCrO₄ be explained by Le Châtelier’s principle? Adding Pb²⁺ ion to the solubility equilibrium stresses the equilibrium. To relieve the stress, the equilibrium shifts to the left to form more solid PbCrO₄.

The common ion effect also plays a role in the use of barium sulfate (BaSO₄) when X rays are taken of the digestive system. Recall that patients who need such X rays must drink a mixture containing BaSO₄. The low solubility of BaSO₄ helps ensure that the amount of the toxic barium ion absorbed into patient’s system is small enough to be harmless. The procedure is further safeguarded by the addition of sodium sulfate (Na₂SO₄), a soluble ionic compound that provides a common ion, SO₄²⁻. How does the additional SO₄²⁻ affect the concentration of barium ion in the mixture that patients must drink?
BaSO₄(s) ⇌ Ba²⁺(aq) + SO₄²⁻(aq)

Le Châtelier’s principle predicts that additional SO₄²⁻ from the Na₂SO₄ will shift the equilibrium to the left to produce more solid BaSO₄ and reduce the number of harmful Ba²⁺ ions in solution.

**Solubility Equilibria in the Laboratory**

Suppose you are given a clear aqueous solution and told that it could contain almost any of the ions formed by the common metallic elements. Your job is to find out which ions the solution contains. It seems an impossible task, but chemists have worked out a scheme that allows you to separate and identify many common metal ions. The scheme is based upon the differing solubilities of the ions. For example, only three ions form insoluble chlorides—Ag⁺, Pb²⁺, and Hg₂²⁺. That may give you an idea of how you could proceed. If you add HCl to your solution, those three ions will precipitate as a white solid that would contain AgCl(s), PbCl₂(s), and Hg₂Cl₂(s) if all three of the metal ions are present. The ions that do not form insoluble chlorides will be left in the clear solution, which can be separated from the white solid.

Is PbCl₂ present in the white solid? To find out, you can use the fact that PbCl₂ is soluble in hot water but AgCl and Hg₂Cl₂ are not. After adding water and heating the mixture of chlorides, you can separate the liquid (that could contain dissolved PbCl₂) from AgCl and Hg₂Cl₂, which are still in solid form. What test could you use to show that the separated liquid contains Pb²⁺? Remember that PbCrO₄ is a sparingly soluble compound. You could add K₂CrO₄. If a yellow precipitate of PbCrO₄ forms, you have proof of the presence of Pb²⁺.

Are Hg₂Cl₂, or AgCl, or both compounds present in the remaining precipitate? Silver chloride is soluble in ammonia; mercury(I) chloride forms a black precipitate with ammonia. Suppose you add ammonia and get a black precipitate. You can conclude that Hg₂²⁺ is present. But did AgCl dissolve in ammonia, or was all the white precipitate Hg₂Cl₂? To find out, you can add HCl to the solution. HCl will interact with the ammonia in the solution and cause AgCl to precipitate again.

The procedure you have been reading is just one step in the complete analysis of the initial solution of ions. **Figure 18-18** is a flow chart of the steps in the procedure. Follow the chart step by step through each identification. Note that as each reactant is added (HCl, hot water, NH₃), a separation is made between those ions that are soluble and those that are not.

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**Section 18.3 Assessment**

20. List the information you would need in order to calculate the concentration of a product in a reaction mixture at equilibrium.

21. How can you use the solubility product constant to calculate the solubility of a sparingly soluble ionic compound?

22. What is a common ion? Explain how a common ion reduces the solubility of an ionic compound.

23. **Thinking Critically** When aqueous solutions of two ionic compounds are mixed, how does $Q_{sp}$ relate to $K_{sp}$ for a possible precipitate?

24. **Designing an Experiment** An aqueous solution is known to contain either Mg²⁺ or Pb²⁺. Design an experiment based on solubilities that would help you determine which of the two ions is present. The solubilities of many ionic compounds are given in **Table C-10** in Appendix C.
Comparing Two Solubility Product Constants

Le Châtelier’s principle is a powerful tool for explaining how a reaction at equilibrium shifts when a stress is placed on the system. In this experiment, you can use Le Châtelier’s principle to evaluate the relative solubilities of two precipitates. By observing the formation of two precipitates in the same system, you can infer the relationship between the solubilities of the two ionic compounds and the numerical values of their solubility product constants ($K_{sp}$). You will be able to verify your own experimental results by calculating the molar solubilities of the two compounds using the $K_{sp}$ for each compound.

**Problem**

How can a saturated solution of one ionic compound react with another ionic compound to form another precipitate? What is the relationship between solubility and the $K_{sp}$ value of a saturated solution?

**Objectives**

- **Observe** evidence that a precipitate is in equilibrium with its ions in solution.
- **Infer** the relative solubilities of two sparingly soluble ionic compounds.
- **Compare** the values of the $K_{sp}$ for two different compounds and relate them to your observations.
- **Explain** your observations of the two precipitates by using Le Châtelier’s principle.
- **Calculate** the molar solubilities of the two ionic compounds from their $K_{sp}$ values.

**Materials**

- AgNO₃ solution
- NaCl solution
- Na₂S solution
- 24-well microplate
- thin-stem pipettes (3)
- wash bottle

**Materials**

- AgNO₃ solution
- NaCl solution
- Na₂S solution
- 24-well microplate
- thin-stem pipettes (3)
- wash bottle

**Safety Precautions**

- Always wear safety goggles, gloves, and a lab apron.
- Silver nitrate is highly toxic and will stain skin and clothing.

**Pre-Lab**

1. Read the entire CHEMLAB.
2. Prepare all written materials that you will take into the laboratory. Be sure to include safety precautions, procedure notes, and a data table in which to record your observations.

**Precipitate Formation**

<table>
<thead>
<tr>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Step 3</td>
</tr>
<tr>
<td>Step 5</td>
</tr>
<tr>
<td>Step 6</td>
</tr>
</tbody>
</table>

**Procedure**

1. Place 10 drops of AgNO₃ solution in well A1 of a 24-well microplate. Place 10 drops of the same solution in well A2.
2. Add 10 drops of NaCl solution to well A1 and 10 drops to well A2.
3. Allow the precipitate to form in each well. Record your observations.

4. To well A2, add 10 drops of Na₂S solution.

5. Allow the precipitate to form. Record your observations of the precipitate.

6. Compare the contents of wells A1 and A2 and record your observations in your table.

**Cleanup and Disposal**

1. Use a wash bottle to transfer the contents of the well plate into a large waste beaker.

2. Wash your hands thoroughly after all lab work and cleanup are complete.

**Analyze and Conclude**

1. **Analyzing Information** Write the complete equation for the double-replacement reaction that occurred when NaCl and AgNO₃ were mixed in wells A1 and A2 in step 2. Write the net ionic equation.

2. **Analyzing Information** Write the solubility product constant expression for the equilibrium established in wells A1 and A2 in step 2.  
   \[ K_{sp} (\text{AgCl}) = 1.8 \times 10^{-10} \]

3. **Analyzing Information** Write the equation for the equilibrium that was established in well A2 when you added Na₂S.  
   \[ K_{sp} (\text{Ag}_2\text{S}) = 8 \times 10^{-48} \]

4. **Inferring** Identify the two precipitates by color.

5. **Comparing and Contrasting** Compare the \( K_{sp} \) values for the two precipitates. Infer which of the two ionic compounds is more soluble.

6. **Recognizing Cause and Effect** Use Le Châtelier’s principle to explain how the addition of Na₂S in procedure step 4 affected the equilibrium established in well A2.

7. **Using Numbers** Calculate the molar solubilities of the two precipitates using the \( K_{sp} \) values. Which of the precipitates is more soluble?

8. **Thinking Critically** What evidence from this experiment supports your answer to question 7? Explain.

9. **Error Analysis** Did you observe the well plate from the side as well as from the top? What did you notice?

10. **Developing General Rules** The solubility of an ionic compound depends upon the nature of the cations and anions that make up the compound. The reactants you used in this CHEMLAB are all soluble ionic compounds, whereas, the precipitates are insoluble. How does soluble Na₂S differ from insoluble Ag₂S? How does soluble NaCl differ from insoluble AgCl? Use this information and \( K_{sp} \) data from Table 18-3 and the Handbook of Chemistry and Physics to develop general rules for solubility. What group of metal ions is not found in sparingly soluble compounds? What polyatomic ions, positive and negative, form only soluble ionic compounds? How does \( K_{sp} \) relate to a compound’s relative solubility?

**Real-World Chemistry**

1. Research how industries use precipitation to remove hazardous chemicals from wastewater before returning it to the water cycle.

2. *Hard water* is the name given to water supplies that contain significant concentrations of Mg²⁺ and Ca²⁺ ions. Check on the solubility of ionic compounds formed with these ions and predict what problems they may cause.

3. Explain what would happen if you lost the stopper for a bottle of a saturated solution of lead sulfate (PbSO₄) and the bottle stood open to the air for a week. Would your answer be different if it were an unsaturated solution? Explain.
The Haber Process

Diatomeric nitrogen makes up about 79 percent of Earth’s atmosphere. A few species of soil bacteria can use atmospheric nitrogen to produce ammonia (NH₃). Other species of bacteria then convert the ammonia into nitrite and nitrate ions, which can be absorbed and used by plants. Ammonia also can be synthesized.

Applying Le Châtelier’s Principle

The process of synthesizing large amounts of ammonia from nitrogen and hydrogen gases was first demonstrated in 1909 by Fritz Haber, a German research chemist, and his English research assistant, Robert LeRossignol. The Haber process involves this reaction.

\[ \text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g) + \text{heat} \]

The process produces high yields of ammonia by manipulating three factors that influence the reaction—pressure, temperature, and catalytic action.

During the synthesis of ammonia, four molecules of reactant produce two molecules of product. According to Le Châtelier’s principle, if the pressure on this reaction is increased, the forward reaction will speed up to reduce the stress because two molecules exert less pressure than four molecules. Increased pressure will also cause the reactants to collide more frequently, thus increasing the reaction rate. Haber’s apparatus used a pressure of 2 × 10^5 kPa.

The forward reaction is favored by a low temperature because the stress caused by the heat generated by the reaction is reduced. But low temperature decreases the number of collisions between reactants, thus decreasing the rate of reaction. Haber compromised by using an intermediate temperature of about 450°C.

A catalyst is used to decrease the activation energy and thus, increase the rate at which equilibrium is reached. Haber used iron as a catalyst in his process.

The Industrial Process

Haber’s process incorporated several operations that increased the yield of ammonia. The reactant gases entering the chamber were warmed by heat produced by the reaction. The reactant-product mixture was allowed to cool slowly after reacting over the catalyst. Ammonia gas was removed from the process by liquefaction, and the unreacted nitrogen and hydrogen were recycled back into the process.

Carl Bosch, a German industrial chemist improved Haber’s process by designing new reaction chambers, improving pressurizing pumps, and finding inexpensive catalysts. By 1913, Bosch had built the first plant for synthesizing ammonia in Oppau, Germany.

Investigating the Technology

1. **Thinking Critically** How does removing ammonia from the process affect equilibrium?

2. **Using Resources** Research how ammonia is used in the production of many agricultural fertilizers and other products.

Visit the Chemistry Web site at chemistrymc.com to find links to more information about the Haber process and ammonia.
Summary

18.1 Equilibrium: A State of Dynamic Balance

- A reversible reaction is one that can take place in both the forward and reverse directions.
- A reversible reaction leads to an equilibrium state in which the forward and reverse reactions take place at equal rates and the concentrations of reactants and products remain constant.
- You can write the equilibrium constant expression for an equilibrium system using the law of chemical equilibrium.
- The equilibrium constant expression is a ratio of the molar concentrations of the products divided by the molar concentrations of the reactants with all concentrations in the ratio raised to a power equal to their coefficients in the balanced chemical equation.
- The value of the equilibrium constant expression, $K_{eq}$, is a constant for a given temperature.
- A large value for $K_{eq}$ means that the products are favored at equilibrium; a small $K_{eq}$ value means that the reactants are favored.
- You can calculate $K_{eq}$ by substituting known equilibrium concentrations into the equilibrium constant expression.

18.2 Factors Affecting Chemical Equilibrium

- Le Châtelier’s principle describes how an equilibrium system shifts in response to a stress or disturbance. A stress is any change in the system at equilibrium.
- An equilibrium can be forced in the direction of the products by adding a reactant or by removing a product. It can be forced in the direction of the reactants by adding a product or removing a reactant.
- When an equilibrium shifts in response to a change in concentration or volume, the equilibrium position changes but $K_{eq}$ remains constant. A change in temperature, however, alters both the equilibrium position and the value of $K_{eq}$.

18.3 Using Equilibrium Constants

- Given $K_{eq}$, the equilibrium concentration of a substance can be calculated if you know the equilibrium concentrations of all other reactants and products.
- The solubility product constant expression, $K_{sp}$, describes the equilibrium between a sparingly soluble ionic compound and its ions in solution.
- You can calculate the molar solubility of an ionic compound using the solubility product constant expression.
- The ion product, $Q_{sp}$, can be calculated from the molar concentrations of the ions in a solution and compared with the $K_{sp}$ to determine whether a precipitate will form when two solutions are mixed.
- The solubility of a substance is lower when the substance is dissolved in a solution containing a common ion. This is called the common ion effect.

Key Equations and Relationships

- $K_{eq} = \frac{[C]^c[D]^d}{[A]^a[B]^b}$ (p. 563)

Vocabulary

- chemical equilibrium (p. 561)
- common ion (p. 584)
- common ion effect (p. 584)
- equilibrium constant (p. 563)
- heterogeneous equilibrium (p. 565)
- homogeneous equilibrium (p. 564)
- law of chemical equilibrium (p. 563)
- Le Châtelier’s principle (p. 569)
- reversible reaction (p. 560)
- solubility product constant (p. 578)
Go to the Chemistry Web site at chemistrymc.com for additional Chapter 18 Assessment.

Concept Mapping

25. Fill in the spaces on the concept map with the following phrases: equilibrium constant expressions, reversible reactions, heterogeneous equilibria, homogeneous equilibria, chemical equilibria.

Mastering Concepts

26. Describe an equilibrium in everyday life that illustrates a state of balance between two opposing processes. (18.1)

27. Given the fact that the concentrations of reactants and products are not changing, why is the word dynamic used for describing chemical equilibrium? (18.1)

28. How can you indicate in a chemical equation that a reaction is reversible? (18.1)

29. Although the general equation for a chemical reaction is reactants $\rightarrow$ products, explain why this equation is not complete for a system at equilibrium. (18.1)

30. Explain the difference between a homogeneous equilibrium and a heterogeneous equilibrium. (18.1)

31. What is an equilibrium position? (18.1)

32. Explain how to use the law of chemical equilibrium in writing an equilibrium constant expression. (18.1)

33. Why does a numerically large $K_{eq}$ mean that the products are favored in an equilibrium system? (18.1)

34. Why should you pay attention to the physical states of all reactants and products when writing equilibrium constant expressions? (18.1)

35. How can an equilibrium system contain small and unchanging amounts of products yet have large amounts of reactants? What can you say about the relative size of $K_{eq}$ for such an equilibrium? (18.1)

36. Describe the opposing processes in the physical equilibrium that exists in a closed container half-filled with liquid ethanol. (18.1)

37. What is meant by a stress on a reaction at equilibrium? (18.2)

38. How does Le Châtelier’s principle describe an equilibrium’s response to a stress? (18.2)

39. Why does removing a product cause an equilibrium to shift in the direction of the products? (18.2)

40. When an equilibrium shifts toward the reactants in response to a stress, how is the equilibrium position changed? (18.2)

41. Use Le Châtelier’s principle to explain how a shift in the equilibrium $\text{H}_2\text{CO}_3(aq) \rightleftharpoons \text{H}_2\text{O}(l) + \text{CO}_2(g)$ causes a soft drink to go flat when its container is left open to the atmosphere. (18.2)

42. How is $K_{eq}$ changed when heat is added to an equilibrium in which the forward reaction is exothermic? Explain using Le Châtelier’s principle. (18.2)

43. Changing the volume of the system alters the equilibrium position of this equilibrium.

$$\text{N}_2(g) + 3\text{H}_2(g) \rightleftharpoons 2\text{NH}_3(g)$$

But a similar change has no effect on this equilibrium.

$$\text{H}_2(g) + \text{Cl}_2(g) \rightleftharpoons 2\text{HCl}(g)$$

Explain. (18.2)

44. How might the addition of a noble gas to the reaction vessel affect this equilibrium?

$$2\text{N}_2\text{H}_4(g) + 2\text{NO}_2(g) \rightleftharpoons 3\text{N}_2(g) + 4\text{H}_2\text{O}(g)$$

Assume that the volume of the reaction vessel does not change. (18.2)

45. When an equilibrium shifts to the right, what happens to the following? (18.2)

a. the concentrations of the reactants
b. the concentrations of the products

46. How would each of the following changes affect the equilibrium position of the system used to produce methanol from carbon monoxide and hydrogen? (18.2)

$$\text{CO}(g) + 2\text{H}_2(g) \rightleftharpoons \text{CH}_3\text{OH}(g) + \text{heat}$$

a. adding CO to the system
b. cooling the system
c. adding a catalyst to the system
d. removing CH$_3$OH from the system
e. decreasing the volume of the system
47. Why is the concentration of a solid not included as part of the solubility product constant? (18.3)

48. What does it mean to say that two solutions have a common ion? Give an example that supports your answer. (18.3)

49. Explain the difference between $Q_{sp}$ and $K_{sp}$. (18.3)

50. Explain why a common ion lowers the solubility of an ionic compound. (18.3)

51. Describe the solution that results when two solutions are mixed and $Q_{sp}$ is found to equal $K_{sp}$. Does a precipitate form?

Mastering Problems  

The Equilibrium Constant Expression (18.1)

52. Write equilibrium constant expressions for these homogeneous equilibria.

a. $2N_2H_4(g) + 2NO_2(g) \rightleftharpoons 3N_2(g) + 4H_2O(g)$

b. $2NaCl(s) \rightleftharpoons Na^+(aq) + Cl^-(aq)$

c. $I_2(g) \rightleftharpoons 2I(g)$

d. $2SO_3(g) + CO_2(g) \rightleftharpoons CS_2(g) + 4O_2(g)$

53. Write equilibrium constant expressions for these heterogeneous equilibria.

a. $2NaHCO_3(s) \rightleftharpoons Na_2CO_3(s) + H_2O(g) + CO_2(g)$

b. $C_6H_6(l) \rightleftharpoons C_6H_6(g)$

c. $Fe_2O_3(s) + 4H_2(g) \rightleftharpoons 3Fe(s) + 4H_2O(g)$

54. Pure water has a density of 1.00 g/mL at 297 K. Calculate the molar concentration of pure water at this temperature.

55. Calculate $K_{eq}$ for the following equilibrium when $[SO_3] = 0.0160 \text{ mol/L}$, $[SO_2] = 0.00560 \text{ mol/L}$, and $[O_2] = 0.00210 \text{ mol/L}$.

\[2SO_3(g) \rightleftharpoons 2SO_2(g) + O_2(g)\]

56. $K_{eq}$ for this reaction is 3.63.

$A + 2B \rightleftharpoons C$

The data in the table shows the concentrations of the reactants and product in two different reaction mixtures at the same temperature. Does the data provide evidence that both reactions are at equilibrium?

<table>
<thead>
<tr>
<th>Concentrations of A, B, and C</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (mol/L)</td>
</tr>
<tr>
<td>0.500</td>
</tr>
<tr>
<td>0.250</td>
</tr>
</tbody>
</table>

57. When solid ammonium chloride is put in a reaction vessel at 323 K, the equilibrium concentrations of both ammonia and hydrogen chloride are found to be $0.0660 \text{ mol/L}$. $NH_4Cl(s) \rightleftharpoons NH_3(g) + HCl(g)$. Calculate $K_{eq}$.

58. Suppose you have a cube of pure manganese metal measuring 5.25 cm on each side. You find that the mass of the cube is 1076.6 g. What is the molar concentration of manganese in the cube?

Le Châtelier’s Principle (18.2)

59. Use Le Châtelier’s principle to predict how each of the following changes would affect this equilibrium. $H_2(g) + CO_2(g) \rightleftharpoons H_2O(g) + CO(g)$

a. adding $H_2O(g)$ to the system

b. removing $CO(g)$ from the system

c. adding $H_2(g)$ to the system

d. adding something to the system to absorb $CO_2(g)$

60. How would increasing the volume of the reaction vessel affect these equilibria?

a. $NH_4Cl(s) \rightleftharpoons NH_4^+(aq) + Cl^-(aq)$

b. $N_2(g) + O_2(g) \rightleftharpoons 2NO(g)$

61. How would decreasing the volume of the reaction vessel affect these equilibria?

a. $2N_2H_4(g) + 2NO_2(g) \rightleftharpoons 3N_2(g) + 4H_2O(g)$

b. $2H_2O(g) \rightleftharpoons 2H_2(g) + O_2(g)$

62. How would these equilibria be affected by increasing the temperature?

a. $4NH_3(g) + 5O_2(g) \rightleftharpoons 4NO(g) + 6H_2O(g)$

b. $heat + NaCl(s) \rightleftharpoons Na^+(aq) + Cl^-(aq)$

63. Ethylene ($C_2H_4$) reacts with hydrogen to form ethanol ($C_2H_5OH$).

$C_2H_4(g) + H_2(g) \rightleftharpoons C_2H_5OH(g) + heat$

How would you regulate the temperature of this equilibrium in order to do the following?

a. increase the yield of ethanol

b. decrease the concentration of ethylene

c. increase the amount of hydrogen in the system

64. How would simultaneously decreasing the temperature and volume of the system affect these equilibria?

a. $heat + CaCO_3(s) \rightleftharpoons CaO(s) + CO_2(g)$

b. $4NH_3(g) + 5O_2(g) \rightleftharpoons 4NO(g) + 6H_2O(g) + heat$

Calculations Using $K_{eq}$ (18.3)

65. $K_{eq}$ is 1.60 at 933 K for this reaction. $H_2(g) + CO_2(g) \rightleftharpoons H_2O(g) + CO(g)$

Calculate the equilibrium concentration of hydrogen when $[CO_2] = 0.320 \text{ mol/L}$, $[H_2O] = 0.240 \text{ mol/L}$, and $[CO] = 0.280 \text{ mol/L}.$
66. At 2273 K, \( K_{eq} = 6.2 \times 10^{-4} \) for the reaction
\[ \text{N}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\text{NO}(\text{g}) \]
If \([\text{N}_2] = 0.05200 \text{ mol/L} \) and \([\text{O}_2] = 0.00120 \text{ mol/L} \), what is the concentration of NO at equilibrium?

### Calculations Using \( K_{sp} \) (18.3)

67. Calculate the ion product to determine if a precipitate will form when 125 mL 0.00500 \( \text{M} \) sodium chloride is mixed with 125 mL 0.00100 \( \text{M} \) silver nitrate solution.

68. Calculate the molar solubility of strontium chromate in water at 298 K if \( K_{sp} = 3.5 \times 10^{-5} \).

69. Will a precipitate form when 1.00 L of 0.150 \( \text{M} \) iron(II) chloride solution is mixed with 2.00 L of 0.0333 \( \text{M} \) sodium hydroxide solution? Explain your reasoning and show your calculations.

### Mixed Review

Sharpen your problem-solving skills by answering the following.

70. How many moles per liter of silver chloride will be in a saturated solution of AgCl? \( K_{sp} = 1.8 \times 10^{-10} \)

71. A 6.00-L vessel contains an equilibrium mixture of 0.0222 mol PCl₃, 0.0189 mol PCl₅, and 0.1044 mol Cl₂. Calculate \( K_{eq} \) for the following reaction.

\[ \text{PCl}_5(\text{g}) \rightleftharpoons 3\text{PCl}_3(\text{g}) + \text{Cl}_2(\text{g}) \]

72. How would simultaneously increasing the temperature and volume of the system affect these equilibria?
   a. \( 2\text{O}_3(\text{g}) \rightleftharpoons 3\text{O}_2(\text{g}) \) + heat
   b. heat + \( \text{N}_2(\text{g}) + \text{O}_2(\text{g}) \rightleftharpoons 2\text{NO}(\text{g}) \)

73. The solubility product constant for lead(II) arsenate (\( \text{Pb}_3(\text{AsO}_4)_2 \)), is \( 4.0 \times 10^{-36} \) at 298 K. Calculate the molar solubility of the compound at this temperature.

74. How would these equilibria be affected by decreasing the temperature?
   a. \( 2\text{O}_3(\text{g}) \rightleftharpoons 3\text{O}_2(\text{g}) \) + heat
   b. heat + \( \text{H}_2(\text{g}) + \text{F}_2(\text{g}) \rightleftharpoons 2\text{HF}(\text{g}) \)

### Thinking Critically

75. **Predicting** Suppose you’re thinking about using the following reaction to produce hydrogen from hydrogen sulfide.

\[ 2\text{H}_2\text{S}(\text{g}) + \text{heat} \rightleftharpoons 2\text{H}_2(\text{g}) + \text{S}_2(\text{g}) \]

Given that \( K_{eq} \) for the equilibrium is \( 2.27 \times 10^{-4} \), would you expect a high yield of hydrogen? Explain how you could regulate the volume of the reaction vessel and the temperature to increase the yield.

### Writing in Chemistry

79. Research the role that solubility plays in the formation of kidney stones. Find out what compounds are found in kidney stones and their \( K_{sp} \) values. Summarize your findings in a report.

80. The presence of magnesium and calcium ions in water makes the water “hard.” Explain in terms of solubility why the presence of these ions is often undesirable. Find out what measures can be taken to eliminate them.

### Cumulative Review

Refresh your understanding of previous chapters by answering the following.

81. How are electrons shared differently in \( \text{H}_2 \), \( \text{O}_2 \), and \( \text{N}_2 \)? (Chapter 9)

82. How can you tell if a chemical equation is balanced? (Chapter 10)

83. What mass of carbon must burn to produce 4.56 L \( \text{CO}_2 \) gas at STP? (Chapter 14)

\[ \text{C}(\text{s}) + \text{O}_2(\text{g}) \rightarrow \text{CO}_2(\text{g}) \]

84. When you reverse a thermochemical equation, why must you change the sign of \( \Delta H \)? (Chapter 16)
Use these questions and the test-taking tip to prepare for your standardized test.

1. A system reaches chemical equilibrium when
   a. no new product is formed by the forward reaction.
   b. the reverse reaction no longer occurs in the system.
   c. the concentration of reactants in the system is equal to the concentration of products.
   d. the rate at which the forward reaction occurs equals the rate of the reverse reaction.

2. What does a value of $K_{eq}$ greater than 1 mean?
   a. more reactants than products exist at equilibrium
   b. more products than reactants exist at equilibrium
   c. the rate of the forward reaction is high at equilibrium
   d. the rate of the reverse reaction is high at equilibrium

3. The hydrogen sulfide produced as a byproduct of petroleum refinement can be used to produce elemental sulfur: $2H_2S(g) + SO_2(g) \rightarrow 3S(l) + 2H_2O(g)$
   What is the equilibrium constant expression for this reaction?
   a. $K_{eq} = \frac{[H_2O]}{[H_2S][SO_2]}$
   b. $K_{eq} = \frac{[H_2S]^2[SO_2]}{[H_2O]}$
   c. $K_{eq} = \frac{[H_2O]^2}{[H_2S][SO_2]}$
   d. $K_{eq} = \frac{[S]^3[H_2O]^2}{[H_2S][SO_2]}$

4. The following system is in equilibrium:
   $2S(s) + 5F_2(g) \rightleftharpoons SF_4(g) + SF_6(g)$
   The equilibrium will shift to the right if _____.
   a. the concentration of SF$_4$ is increased
   b. the concentration of SF$_6$ is increased
   c. the pressure on the system is increased
   d. the pressure on the system is decreased

Interpreting Tables Use the table to answer questions 5–7.

5. What is the $K_{sp}$ for MnCO$_3$?
   a. $2.24 \times 10^{-11}$
   b. $4.00 \times 10^{-11}$
   c. $1.12 \times 10^{-9}$
   d. $5.60 \times 10^{-9}$

6. What is the molar solubility of MnCO$_3$ at 298 K?
   a. $4.73 \times 10^{-6}M$
   b. $6.32 \times 10^{-2}M$
   c. $7.48 \times 10^{-5}M$
   d. $3.35 \times 10^{-5}M$

7. A 50.0-mL volume of $3.00 \times 10^{-6}M$ K$_2$CO$_3$ is mixed with 50.0 mL of MnCl$_2$. A precipitate of MnCO$_3$ will form only when the concentration of the MnCl$_2$ solution is greater than _____.
   a. $1.49 \times 10^{-5}M$
   b. $1.02 \times 10^{-5}M$
   c. $2.99 \times 10^{-5}M$
   d. $7.47 \times 10^{-6}M$

8. Which of the following statements about the common ion effect is NOT true?
   a. The effects of common ions on an equilibrium system can be explained by Le Châtelier’s principle.
   b. The decreased solubility of an ionic compound due to the presence of a common ion is called the common ion effect.
   c. The addition of NaCl to a saturated solution of AgCl will produce the common ion effect.
   d. The common ion effect is due to a shift in equilibrium towards the aqueous products of a system.

9. If the forward reaction of a system in equilibrium is endothermic, increasing the temperature of the system will _____.
   a. shift the equilibrium to the left
   b. shift the equilibrium to the right
   c. decrease the rate of the forward reaction
   d. decrease the rate of the reverse reaction

10. Cl$_2$(g) + 3O$_2$(g) + F$_2$(g) $\rightleftharpoons$ 2ClO$_3$F(g)
    The formation of perchloryl fluoride (ClO$_3$F) from its elements has an equilibrium constant of $3.42 \times 10^{-9}$ at 298 K. If $[Cl_2] = 0.563M$, $[O_2] = 1.01M$, and $[ClO_3F] = 1.47 \times 10^{-5}M$ at equilibrium, what is the concentration of F$_2$?
    a. $9.18 \times 10^6M$
    b. $3.73 \times 10^{-10}M$
    c. $1.09 \times 10^{-1}M$
    d. $6.32 \times 10^{-2}M$

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**Test-Taking Tip**

Maximize Your Score If possible, find out how your standardized test will be scored. In order to do your best, you need to know if there is a penalty for guessing, and if so, what the penalty is. If there is no random-guessing penalty at all, you should always fill in an answer, even if you haven’t read the question!